An Evolutionary Game-Theoretic Approach to Open Access

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Abstract

The paper presents an evolutionary game-theoretic approach to open access publishing as an asymmetric game between scientists and publishers. We show how the ordinary differential equations of the model presented can be written as a system of Hamiltonian partial differential equations. The understanding of the setting as a Hamiltonian system implies some properties reflecting the qualitative behavior of the system.

1 Introduction

The topic of open access publishing has been extensively and controversially discussed. For general information see, for instance, [11, 12, 13]. There are many models of open access named after different colors such as golden or green roads to open access [2, 10], but we do not want to go into more details, here. Concerning scientific publishing particularly in mathematics, we refer to [1, 7] and [9] as well as to the references therein.

In [4] the authors illustrate a game-theoretic approach to open access publishing in order to understand different publication patterns within different scientific disciplines. The underlying observation is that there are communities where open access publishing is widely adopted, whereas other scientific communities are far away from practicing any open access publishing. First, different classical game settings are discussed namely a zero sum game, a game similar to the Prisoners' Dilemma (up to sign), and a stag hunt game version, all describing a Nash equilibrium dilemma of the non-open access communities. Second, these classical settings are transferred into their quantum game extensions which allow to tackle the dilemma mentioned that cannot be solved within the classical setting.

The classical approach in [4] is formulated as a two-player game where the players are authors (scientists) in symmetrical situations. The two authors have the same set $\{s_1, s_2\}$ of strategies at their disposal. Consequently, the game looks

exactly the same to both of its players. For the open access game each player has to decide between the option to publish open access as the first strategy s_1 or, as the second strategy s_2, to conventionally publish in traditional journals where articles go through peer reviewing. Depending on the strategy chosen by the co-player, the players aim to maximize their success of the game. Due to the symmetric situation both players do not only have the same set of strategies but also have the same payoff matrices.

In contrast to the approach of a symmetric two-scientists game, the present paper attempts to explain mathematically the open access play as a conflict of interest between scientists and publishers, as it is emphasized by the rapidly accumulating literature on open access and electronic publishing. Assuming that the open access problem is not a conflict between those scientists who publish open access and those who do not, one takes the view that it is more adequate to understand the problem of open access as an asymmetric conflict between scientists and publishers.

The crucial point of this view is that it actually suggests a bimatrix game describing this asymmetric setting.

We do not explain why both players – scientists and publishers – behave as they do in the game situation described, but we will give a mathematical description of their interaction in the game. Moreover, we do not argue for preferences in choosing one game strategy over another.

2   The Game Setting

We consider scientists and publishers as players in different positions them having different strategy sets as well as different payoff matrices. Moreover, we take into consideration whole populations of players parts of which choose either one or the other strategy.

The set of pure strategies for the scientists is $\{s_1, s_2\}$, where s_1 is publishing open access and s_2 stands for conventional publishing. Furthermore, let $0 \leq x_1 \leq 1$ and $0 \leq x_2 \leq 1$ denote the relative parts of the scientists' population playing strategy s_1 and s_2, respectively. If, for example, half of all scientists publishes open access and the other does not, one has $x_1 = x_2 = \frac{1}{2}$. Or, if the relation is 20% to 80%, then $x_1 = \frac{1}{5}$ and $x_2 = \frac{4}{5}$. Since x_1 and x_2 describe relative frequencies, we see that $x_1 + x_2 = 1$ always holds true. Hence, the set of all possible mixed strategies for the scientists' population is

$$S = \left\{ \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \ : \ 0 \leq x_1, x_2 \leq 1 \text{ and } x_1 + x_2 = 1 \right\}.$$

The opponent population of publishers consists of two types playing pure strategies $\{p_1, p_2\}$. Here strategy p_1 means the publisher accepts or realizes open access

publishing, whereas p_2 represents the strategy of definite declining any way of open access. For the publishers' population let y_i be the frequency of strategy p_i, where $i = 1, 2$. Thus, all possible mixed strategies for the publishers' population are given by the set

$$P = \left\{ \boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2 \ : \ 0 \le y_1, y_2 \le 1 \text{ and } y_1 + y_2 = 1 \right\} .$$

In order to discuss the conflict of open access publishing, we proceed with establishing the payoff matrices A for the scientists and B for the publishers. Each payoff depends on the strategies chosen by the participating players. A player in the scientists' population using strategy s_i against a player from the publishers' population using strategy p_j obtains the payoff a_{ij}, whereas the opponent obtains b_{ji}, with $i, j = 1, 2$.

As reasoned in [4], it is convenient to assume that the scientist tries to maximize his scientific reputation. Let $R > 0$ denote the reputation payoff that the scientist is awarded. This payoff will be slightly reduced by $0 < r < R$ in case of publishing open access. Hence, we still have $R - r > 0$ for the total reputation. In addition, we take into consideration the impact of an article by the scientist. The impact gives some payoff $I > 0$ both for the scientist and for the publisher. Further, this impact obviously will reduce by some $0 < \iota < I$, if the concerning journal is not sufficiently available. Moreover, we assume that $\iota < r$ saying that the reduced impact in case of non-open access publishing is smaller than the loss of reputation in case of open access publishing. Open access publishing will cause expenses $L > 0$ that are shared equally by both players if both play their open access strategy. Furthermore, $G > 0$ expresses some moderate journal price scientists have to pay for library subscriptions as well as the compensation for expenses of the publisher. Finally, let us introduce $P > 0$ for exorbitant profit representing the prices of the most expensive journals coupled with continuing non-open access publishing and taking the tremendously increasing fees for access for granted. For our considerations, we make the realistic assumption that P is so large that the inequality

$$G + P - L > r - \iota \tag{2.1}$$

holds true, which is equivalent to

$$G + P > L + r - \iota .$$

We already have $r - \iota > 0$ and the open access expenses L actually cannot exceed the upper bound given by this inequality.

If an open access publishing scientist meets a publisher accepting open access, the scientist will get his reduced reputation, his research impact, and will pay for maintaining some open access publishing system as well as for the journal subscription. The publisher takes the price for the journal subscription and some amount given by the impact. Apart from this, the publisher contributes to some

extent to open access. Hence, the payoff is $(R-r) + I - \dfrac{L}{2} - G$ for the scientist and $G + I - \dfrac{L}{2}$ for the publisher.

If the scientist's strategy is open access whereas the publisher plays the non-open access strategy, the scientist's payoff will be $(R-r) + I - L$. The scientists have to run the open access completely by themselves and do not have any expenses for journal subscriptions. For the publisher nothing happens and hence the payoff is 0.

A non-open access publishing scientist gets his full reputation R but only reduced research impact $I - \iota$. Additionally, the journal has to be paid, which reduces the payoff by G. If the publisher's counter-strategy is open access, the payoff for the scientist will be $R + (I - \iota) - G$. In this case the payoff for the publisher will be $G + (I - \iota) - L$ paying the total sum for open access acceptance. Moreover, the payoff is $R + (I - \iota) - G - P$ for the scientist and $G + (I - \iota) + P$ for the publisher in case of the non-open access strategy of the publisher.

Altogether, we have the complete income-and-loss statement as given in Table 1.

Table 1: Complete payoffs

strategies	payoff for the scientist	payoff for the publisher
$s_1 \longleftrightarrow p_1$	$(R-r) + I - \dfrac{L}{2} - G$	$G + I - \dfrac{L}{2}$
$s_1 \longleftrightarrow p_2$	$(R-r) + I - L$	0
$s_2 \longleftrightarrow p_1$	$R + (I - \iota) - G$	$G + (I - \iota) - L$
$s_2 \longleftrightarrow p_2$	$R + (I - \iota) - G - P$	$G + (I - \iota) + P$

Hence, the corresponding payoff matrices are

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} (R-r) + I - \dfrac{L}{2} - G & (R-r) + I - L \\ R + (I - \iota) - G & R + (I - \iota) - G - P \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} G + I - \dfrac{L}{2} & G + (I - \iota) - L \\ 0 & G + (I - \iota) + P \end{pmatrix}.$$

Now, both payoff matrices determine the payoffs for the entire populations. For a general description of how this happens we warmly recommend the book [5] by Hofbauer and Sigmund. The corresponding strategies are spread within both populations in accordance with the frequencies of the strategies. Hence, if the scientists' population is in state $x \in S$ and the publishers' population is in state $y \in P$, then the payoff for the entire population of scientists will be

$$\langle x, Ay \rangle = x^T Ay$$

and that for the publishers' population will be

$$\langle y, Bx \rangle = y^T Bx = x^T B^T y \ .$$

As already mentioned, players will choose their game strategies with the intention of maximizing their average payoff. This would be easy if one knew the strategy that the opponent player is going to choose. In case both players of the game choose simultaneously strategies of best reply to the choices of the others, a pair of strategies occurs where both players are incited to keep going into the same direction. Such a pair of strategy choices is known as Nash equilibrium. This is technically expressed by the following definition.

Definition 2.1 *A Nash equilibrium is a pair* $(\overline{x}, \overline{y}) \in S \times P$ *such that*

$$x^T A \overline{y} \leq \overline{x}^T A \overline{y} \quad \text{for all } x \in S$$

as well as

$$y^T B \overline{x} \leq \overline{y}^T B \overline{x} \quad \text{for all } y \in P$$

is fulfilled.

The following considerations aim to give a Nash equilibrium for our open access game setting.

Lemma 2.2 *Each of the differences* $a_{12} - a_{22}$, $a_{21} - a_{11}$, $b_{22} - b_{21}$, *and* $b_{11} - b_{12}$ *is positive.*

Proof. Indeed,

$$a_{12} - a_{22} = -r - L + \iota + G + P > 0$$

by (2.1) and

$$a_{21} - a_{11} = -\iota + r + \frac{L}{2} > 0$$

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by $r > \iota$. Furthermore,

$$b_{22} - b_{21} = G + I - \iota + P$$

which is positive since $I > \iota$. Finally,

$$b_{11} - b_{12} = \frac{L}{2} + \iota$$

which is obviously positive. $\qquad\square$

Lemma 2.3 *Let $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}) \in S \times P$ be given by*

$$\overline{\boldsymbol{x}} = \begin{pmatrix} x_0 \\ 1 - x_0 \end{pmatrix} \quad and \quad \overline{\boldsymbol{y}} = \begin{pmatrix} y_0 \\ 1 - y_0 \end{pmatrix},$$

where

$$x_0 = \frac{b_{22} - b_{12}}{b_{22} - b_{12} + b_{11} - b_{21}} \quad and \quad y_0 = \frac{a_{12} - a_{22}}{a_{12} - a_{22} + a_{21} - a_{11}}.$$

Then

$$\boldsymbol{x}^T A \overline{\boldsymbol{y}} = \overline{\boldsymbol{x}}^T A \overline{\boldsymbol{y}} \quad for\ all\ \boldsymbol{x} \in S$$

and

$$\boldsymbol{y}^T B \overline{\boldsymbol{x}} = \overline{\boldsymbol{y}}^T B \overline{\boldsymbol{x}} \quad for\ all\ \boldsymbol{y} \in P$$

hold true.

Proof. First, by Lemma 2.2 both denominators of the fractions defining x_0 and y_0 are positive. Then we have

$$
\begin{aligned}
A\overline{\boldsymbol{y}} &= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_0 \\ 1 - y_0 \end{pmatrix} \\
&= \begin{pmatrix} y_0(a_{11} - a_{12}) + a_{12} \\ y_0(a_{21} - a_{22}) + a_{22} \end{pmatrix} = \frac{\det A}{a_{12} - a_{22} + a_{21} - a_{11}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\end{aligned}
$$

as well as

$$
\begin{aligned}
B\overline{\boldsymbol{x}} &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ 1 - x_0 \end{pmatrix} \\
&= \begin{pmatrix} x_0(b_{11} - b_{12}) + b_{12} \\ x_0(b_{21} - b_{22}) + b_{22} \end{pmatrix} = \frac{-\det B}{b_{22} - b_{12} + b_{11} - b_{21}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\end{aligned}
$$

Hence,

$$\boldsymbol{x}^T A \overline{\boldsymbol{y}} = \frac{\det A}{a_{12} - a_{22} + a_{21} - a_{11}}(x + (1 - x)) = \frac{\det A}{a_{12} - a_{22} + a_{21} - a_{11}}$$

does not depend on $\boldsymbol{x} = \begin{pmatrix} x \\ 1 - x \end{pmatrix}$ and

$$\boldsymbol{y}^T B \overline{\boldsymbol{x}} = \frac{\det B}{b_{12} - b_{22} + b_{21} - b_{11}} (y + (1 - y)) = \frac{\det B}{b_{12} - b_{22} + b_{21} - b_{11}}$$

is independent of $\boldsymbol{y} = \begin{pmatrix} y \\ 1 - y \end{pmatrix}$. $\square$

Corollary 2.4 *The pair $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}) \in S \times P$ given in Lemma 2.3 is a Nash equilibrium.* $\square$

Remark 2.5 For this game there is no strict, i.e. given by pure strategies, Nash equilibrium. The Nash equilibrium $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}) \in S \times P$ specified in Lemma 2.3 is a unique mixed Nash equilibrium and determined by the following equations, c.f. [5], Chapter 10.2 (Page 116, above),

$$\begin{aligned} a_{11}y_0 + a_{12}(1 - y_0) &= a_{21}y_0 + a_{22}(1 - y_0) \\ b_{11}x_0 + b_{12}(1 - x_0) &= b_{21}x_0 + b_{22}(1 - x_0) \,. \end{aligned}$$

$\square$

3   The Dynamics of the Game

Due to the payoffs of the game, there is an intrinsic dynamics within the system. Actually, starting for instance with the situation that the scientist has chosen strategy s_1 whereas the publisher's strategy is p_1, the outcome is

$$(R - r) + I - \frac{L}{2} - G$$

for the scientist and

$$G + I - \frac{L}{2}$$

for the publisher. In this situation the scientist could increase his payoff by changing his strategy to s_2, since $r - \iota > 0$ and $L > 0$. Indeed, the inequality

$$R - r + I - \frac{L}{2} - G < R + I - \iota - G$$

is equivalent to

$$-\frac{L}{2} < r - \iota \,,$$

which holds true obviously for $r - \iota > 0$ and $L > 0$. After this change in the scientist's strategy the payoff is

$$R + I - \iota - G$$

for the scientist. Holding strategy p_1 the publisher's payoff is

$$G + (I - \iota) - L .$$

In this situation, the publisher sees the chance to change strategy to p_2. The reason is that the publisher's payoff of $G + (I - \iota) - L$ in the game $s_2 \leftrightarrow p_1$ can be increased to $G + (I - \iota) + P$ in a game $s_2 \leftrightarrow p_2$. In fact, $L > 0$ and $P > 0$ give

$$G + (I - \iota) - L < G + (I - \iota) + P .$$

However, finding oneself in the situation of the game $s_2 \leftrightarrow p_2$ the scientist notices that G and P are unnecessary costs, L would not be so huge and the loss of reputation will be moderate. Hence, the scientist will prefer to choose strategy s_1. Altogether, changing strategies in order to maximize outcome ends up moving in a circle.

The dynamics for pairs of pure strategies looks like

$$
\begin{array}{ccc}
(s_1, p_1) & \leftarrow & (s_1, p_2) \\
\downarrow & & \uparrow \\
(s_2, p_1) & \rightarrow & (s_2, p_2)
\end{array}
$$

where the horizontal arrows picture the change of the publishers' strategies and the vertical arrows illustrate the deviation in the scientists' strategies.

This dealing with an "oscillating" system requires a non-static approach. Therefore, we consider the frequencies of the strategies within both populations as time-dependent quantities which are differentiable functions of $t \in \mathbb{R}$, i.e. $x_i = x_i(t)$ and $y_j = y_j(t)$ for $i, j = 1, 2$. The first derivatives of these functions

$$\dot{x}_i = \frac{\mathrm{d}x_i}{\mathrm{d}t} \quad \text{and} \quad \dot{y}_j = \frac{\mathrm{d}y_j}{\mathrm{d}t} \quad \text{for } i, j = 1, 2$$

describe the rate of growth of the respective frequencies.

Modeling the dynamics and the cyclic behavior of that system, we follow the ideas of Hofbauer and Sigmund developed in [5], Chapter 10.3. The increase of strategy s_i of the scientists' population is given by the per capita growth rate $\dfrac{\dot{x}_i}{x_i}$ and equals the difference between its payoff $(A\boldsymbol{y})_i$ and the average payoff $\boldsymbol{x}^T A \boldsymbol{y}$. The same holds true for the publishers' population. This leads to the system of ordinary differential equations

$$
\begin{aligned}
\dot{x}_i &= x_i \left((A\boldsymbol{y})_i - \boldsymbol{x}^T A \boldsymbol{y} \right) \\
\dot{y}_j &= y_j \left((B^T \boldsymbol{x})_j - \boldsymbol{y}^T B^T \boldsymbol{x} \right)
\end{aligned}
$$

for $i, j = 1, 2$.

Let us look closer at these differential equations. We have

$$A\boldsymbol{y} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11}y_1 + a_{12}y_2 \\ a_{21}y_1 + a_{22}y_2 \end{pmatrix}$$

and

$$B^T \boldsymbol{x} = \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_{11}x_1 + b_{21}x_2 \\ b_{12}x_1 + b_{22}x_2 \end{pmatrix} .$$

Hence,

$$(A\boldsymbol{y})_i - \boldsymbol{x}^T A \boldsymbol{y} = y_1(a_{i1} - x_1 a_{11} - x_2 a_{21}) + y_2(a_{i2} - x_1 a_{12} - x_2 a_{22})$$

and

$$(B^T \boldsymbol{x})_j - \boldsymbol{y}^T B^T \boldsymbol{x} = x_1(b_{1j} - y_1 b_{11} - y_2 b_{12}) + x_2(b_{2j} - y_1 b_{21} - y_2 b_{22}) .$$

Since $x_1 + x_2 = 1$ and $y_1 + y_2 = 1$, it is admissible to introduce new variables $x := x_1$ and $y := y_1$ (then $x_2 = 1 - x$ and $y_2 = 1 - y$) and consider the corresponding differential equations for x and y. These are

$$
\begin{aligned}
\dot{x} &= \dot{x}_1 \\
&= x_1\left((A\boldsymbol{y})_1 - \boldsymbol{x}^T A \boldsymbol{y}\right) \\
&= x\left(y_1(a_{11} - x_1 a_{11} - x_2 a_{21}) + y_2(a_{12} - x_1 a_{12} - x_2 a_{22})\right) \\
&= x\left(y(a_{11} - x a_{11} - (1-x)a_{21}) + (1-y)(a_{12} - x a_{12} - (1-x)a_{22})\right) \\
&= x(1-x)\left(y(a_{11} - a_{21}) + (1-y)(a_{12} - a_{22})\right) \\
&= x(1-x)\left((a_{12} - a_{22}) - (a_{12} - a_{22} + a_{21} - a_{11})y\right)
\end{aligned}
$$

and similarly

$$
\dot{y} = \dot{y}_1 = y(1-y)\left(-(b_{22} - b_{21}) + (b_{22} - b_{21} + b_{11} - b_{12})x\right) .
$$

Now, let us introduce new constants by

$$
\begin{aligned}
a &:= a_{12} - a_{22} \\
b &:= a_{21} - a_{11} \\
c &:= b_{22} - b_{21} \\
d &:= b_{11} - b_{12} ,
\end{aligned}
$$

which are positive by Lemma 2.2. Hence, only two equations are remaining namely

$$
\begin{aligned}
\dot{x} &= x(1-x)(a - (a+b)y) & (3.1) \\
\dot{y} &= y(1-y)(-c + (c+d)x) . & (3.2)
\end{aligned}
$$

This is an adequate form for investigating the qualitative behavior of the system.

The subsequent section is devoted to a further investigation of the dynamics of the system given by Equations (3.1) and (3.2).

4 Symplectic Reformulation

We consider the square $Q = \{(x,y) \in \mathbb{R}^2 \ : \ 0 \le x, y \le 1\}$. Obviously, $Q \cong S \times P$ by

$$(x,y) \in Q \quad \mapsto \quad \left(\begin{pmatrix} x \\ 1-x \end{pmatrix}, \begin{pmatrix} y \\ 1-y \end{pmatrix} \right) \in S \times P \ .$$

We use symplectic techniques for investigating what happens in the interior $\mathrm{int}Q$ of Q. More precisely, in order to prove that the orbits of the System (3.1) and (3.2) are closed and do not leave $\mathrm{int}Q$ we describe the whole setting as a Hamiltonian system.

The interior of Q is the open subset

$$M = \{(x,y) \in \mathbb{R}^2 \ : \ 0 < x, y < 1\}$$

of $\mathbb{R}^2$. The function $\varphi : M \to \mathbb{R}$ given by $\varphi(x,y) = xy(1-x)(1-y)$ is positive. Hence,

$$\omega = \frac{1}{\varphi}\mathrm{d}x \wedge \mathrm{d}y$$

defines a 2-form ω on M.

Since we are in dimension 2 the following is clear.

Lemma 4.1 *The pair (M, ω) is a symplectic manifold.* $\qquad\square$

We are going to derive that Equations (3.1) and (3.2) are the equations of motion of the Hamiltonian system (M, ω, H) where the Hamiltonian $H : M \to \mathbb{R}$ is defined as

$$H(x,y) = c\ln(x) + d\ln(1-x) + a\ln(y) + b\ln(1-y) \qquad \text{for } (x,y) \in M \ .$$

The notion of Hamiltonian systems is well-known and widely used in mathematical physics to describe the behavior of systems in classical mechanics. In physics the Hamiltonian H describes the energy of the system under consideration. Hence, it would be interesting to understand, what intrinsic property is determined by the "energy" of open access publishing.

Lemma 4.2 *The Hamiltonian vector field of (M, ω, H) is given by*

$$X_H = \begin{pmatrix} \varphi\dfrac{\partial H}{\partial y} \\[2ex] -\varphi\dfrac{\partial H}{\partial x} \end{pmatrix} \ .$$

Proof. Generally, X_H is defined by the equation

$$\omega(X_H, \ ) = \mathrm{d}H .$$

Clearly,

$$\mathrm{d}H = \frac{\partial H}{\partial x}\mathrm{d}x + \frac{\partial H}{\partial y}\mathrm{d}y .$$

If V is any vector field on M with functions v_1 and v_2 as components, then

$$\omega(V, \ ) = \frac{1}{\varphi}\mathrm{d}x \wedge \mathrm{d}y(V, \ ) = \frac{1}{\varphi}\{v_1\mathrm{d}y - v_2\mathrm{d}x\} .$$

Hence,

$$\omega(V, \ ) = \mathrm{d}H$$

if and only if

$$\frac{\partial H}{\partial x} = -\frac{1}{\varphi}v_2 \quad \text{and} \quad \frac{\partial H}{\partial y} = \frac{1}{\varphi}v_1 ,$$

which proves the assertion. $\qquad\square$

Let us remind that motions of the Hamiltonian System (M, ω, H) are curves $\gamma(t)$ in M satisfying the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\gamma(t)_{|t=s} = X_H(\gamma(s)) .$$

Altogether, we now can prove the following proposition.

Proposition 4.3 *The solutions of (3.1) and (3.2) are exactly the motions of the Hamiltonian system (M, ω, H).*

Proof. We have

$$
\begin{aligned}
\varphi\frac{\partial H}{\partial y} &= xy(1-x)(1-y)\left\{\frac{a}{y} - \frac{b}{1-y}\right\} \\
&= ax(1-x)(1-y) - bxy(1-x) \\
&= x(1-x)(a - ay - by)
\end{aligned}
$$

and

$$
\begin{aligned}
-\varphi\frac{\partial H}{\partial x} &= -xy(1-x)(1-y)\left\{\frac{c}{x} - \frac{d}{1-x}\right\} \\
&= -cy(1-x)(1-y) + dxy(1-y) \\
&= y(1-y)(-c + cx + dx) .
\end{aligned}
$$

Thus, Equations (3.1) and (3.2) are equivalent to

$$\varphi\frac{\partial H}{\partial y} = \dot{x} \quad \text{and} \quad -\varphi\frac{\partial H}{\partial x} = \dot{y} ,$$

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which implies the statement by Lemma 4.2. $\qquad\square$

As a consequence, the solutions in M of the System (3.1) and (3.2) correspond to level set curves of the Hamiltonian H, i.e. the Hamiltonian H is constant along solutions.

Corollary 4.4 *Let $(x(t), y(t))$ give a solution of Equations (3.1) and (3.2). Then*
$$\frac{\mathrm{d}}{\mathrm{d}t} H(x(t), y(t)) = 0 .$$

Proof. Indeed,

$$\frac{\mathrm{d}}{\mathrm{d}t} H(x(t), y(t)) =$$
$$\begin{aligned}
&= \frac{\mathrm{d}}{\mathrm{d}t} \{c \ln(x(t)) + d \ln(1 - x(t)) + a \ln(y(t)) + b \ln(1 - y(t))\} \\
&= c \frac{\dot{x}}{x} - d \frac{\dot{x}}{1 - x} + a \frac{\dot{y}}{y} - b \frac{\dot{y}}{1 - y} \\
&= (c(1 - x) - dx)(a - (a + b)y) + (a(1 - y) - by)(-c + (c + d)x) \\
&= (c - cx - dx)(a - ay - by) + (a - ay - by)(-c + cx + dx) \\
&= 0
\end{aligned}$$

by using (3.1) as well as (3.2) for replacing $\dot{x}$ and $\dot{y}$. $\qquad\square$

In the remainder of this section we want to discuss the shape of the level set curves of H. In particular, we will show that these curves are closed, which implies that the solutions of (3.1) and (3.2) are periodic.

First, we are going to determine the extremal points of H. By
$$\frac{\partial H}{\partial y} = \frac{a}{y} - \frac{b}{1 - y}$$
and
$$\frac{\partial H}{\partial x} = \frac{c}{x} - \frac{d}{1 - x}$$
we obtain
$$\frac{a}{y_0} = \frac{b}{1 - y_0} \quad \text{and} \quad \frac{c}{x_0} = \frac{d}{1 - x_0}$$
as necessary conditions for a critical point (x_0, y_0). This leads to
$$y_0 = \frac{a}{a + b} \quad \text{and} \quad x_0 = \frac{c}{c + d}$$
which corresponds exactly to our Nash equilibrium described in Lemma 2.3. By
$$\begin{aligned}
\frac{\partial^2 H}{\partial^2 x} &= -\frac{c}{x^2} - \frac{d}{(1 - x)^2} , \\
\frac{\partial^2 H}{\partial^2 y} &= -\frac{a}{y^2} - \frac{b}{(1 - y)^2} , \quad \text{and} \\
\frac{\partial^2 H}{\partial x \partial y} = \frac{\partial^2 H}{\partial y \partial x} &= 0 ,
\end{aligned}$$

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we have

$$\frac{\partial^2 H}{\partial^2 x}(x_0, y_0) = -\frac{(c+d)^3}{cd} \quad < \quad 0 \quad \text{and}$$

$$\frac{\partial^2 H}{\partial^2 y}(x_0, y_0) = -\frac{(a+b)^3}{ab} \quad < \quad 0$$

at the point (x_0, y_0). This implies that the matrix

$$\text{Hess} H_{(x_0, y_0)} = \begin{pmatrix} -\dfrac{(c+d)^3}{cd} & 0 \\ 0 & -\dfrac{(a+b)^3}{ab} \end{pmatrix}$$

is negative definite showing that H takes in (x_0, y_0) a maximum as the unique extremum. Second, since

$$\lim_{t \to 0+} \ln(t) = -\infty$$

we have

$$\lim_{(x,y) \to q} H(x, y) = -\infty$$

for each q in the boundary ∂Q of Q. Hence, the graph of the Hamiltonian H looks like a hill over the square Q.
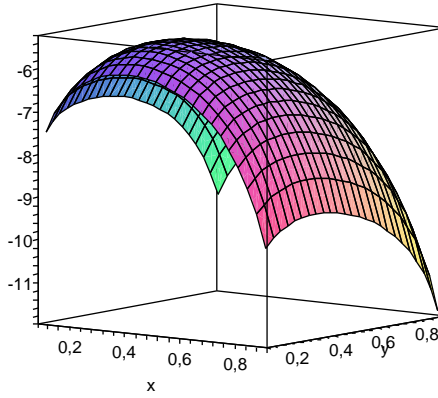
Figure 1: Graph of the Hamiltonian H with parameters $a = 1$, $b = 2$, $c = 2$, and $d = 3$

The top of the hill represents the unique maximum. Towards the boundary ∂Q, the hill crashes into bottomless depths.

Since all solutions of our system correspond to level set curves, they are visualized as contour lines of the hill described. Hence, all orbits surround the point (x_0, y_0) and remain in M.
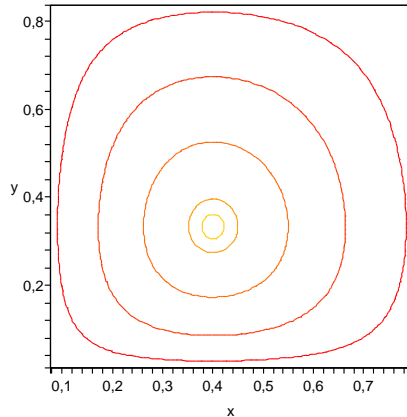
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Figure 2: Periodic orbits surrounding the Nash equilibrium (x_0, y_0) with parameters $a = 1$, $b = 2$, $c = 2$, and $d = 3$

5 Further Discussion

Obviously, the model presented here is a basic approach to discuss the phenomenon of multiple arrangements of realizing open access publishing in different scientific communities. However, in contrast to the approach presented in [4] it treats open access publishing in a canonical way as a game between scientists and publishers. The behavior of oscillation on closed orbits is a rather rough and simplified view of the whole scenario. We do not consider this model to be universally valid, but throughout a certain period of time it seems to allow a reasonably satisfying description of insights which are intuitively clear.

This model can be modified. For instance, taking into account that both the parameter r reducing the scientist's reputation in case of open access publishing and the value of ι expressing the reduced impact in case of non-open access publishing may change in the context of the game. For example, if the rate of open access publishing scientists increases, r will be supposed to decrease etc. Playing a little bit and slightly altering the model presented here may result in further refinements.

Moreover, the huge number of evolutionary game-theoretic approaches in the literature allows a lot of modifications and further developments of these concepts. They vary from biological game theory to concepts that are more applicable in economic contexts. These models are good sources and suitable to establish further improvements in a more fundamental manner, see e.g. [3, 6] and [8] as well as the references therein.

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